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Intergenerational Product Competition and the Incentive to Innovate

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by

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We consider incentives for innovation under various market structures when the new product must compete against an old version for the same customer base. Competition and monopoly in the old product market provide *identical* incentives when (1) the monopolist is protected from new product entry, and (2) innovation is nondrastic in the sense that the monopolist supplies positive quantities of both old and new products. If the monopolist is threatened with entry, monopoly provides *strictly greater* incentives. These results contrast with the familiar finding that competition provides greater incentives for process innovations. Welfare may be greater under monopoly when innovation is valuable.

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1. Introduction

There are two key aspects to the invention of new products. First, product innovations tend to be gradual and build on closely-related older technologies. Second, new products typically do not immediately replace the old versions. Rather, old and new products compete for the same base of customers, even if the new product is far superior in its attributes - as when the new product is more costly and its users do not equally value the product improvements. There are many examples of product innovation, both dramatic and trivial, that possess these features.[1]

Despite the historic significance of this kind of gradual innovation, economists know very little about the connections between intergenerational product competition and the incentive to engage in innovative activity. Theoretical investigations have focussed chiefly on innovations that reduce the cost of producing existing products (e.g. Kamien and Schwartz [1982], Loury [1979]), as opposed to innovations that are embodied directly in new products. A major feature of these process-innovation models is that sellers of the old-generation technology are assumed to play no active role in the market. As we show in this paper, however, the incentive to innovate is crucially affected by rivalry between old- and new-product sellers. Further, traditional conclusions concerning market structure and innovation will no longer hold in the presence of intergenerational product rivalry.

We propose a model in which old and new generations of a product compete for consumers who vary in their valuation of product improvements. The seller of the new product captures consumers with high valuation, but the old product continues to be attractive

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[1] Some rough quantitative evidence on the importance of gradual product improvements is given by Scherer [1980, p. 409]: in a 1977 survey, firms reported that 28% of R&D effort went into development of new products, 59% into improvement of existing products and 13% into reducing the costs of existing manufacturing processes.

to lower-valuation consumers if it is offered at a lower price. The structure of the old product market may be either competitive or monopolistic. Within this framework we ask the question, which market structure maximizes the rents obtainable from the new product, as measured, for example, by the licensing revenue of a patent-holding inventor who controls the new product?

The comparison of market structures hinges on two effects. The *replacement effect*, introduced in Arrow's [1962] analysis of process innovation, holds that innovation is less attractive to a monopolist, since part of the returns from the new product simply replace monopoly rents already earned on the old. We identify a new effect: when the old product is competitively supplied, competition from firms producing the old product reduces the profits of the new product supplier. This *product inertia effect* makes innovation relatively less attractive under competition, since the old product monopolist internalizes this externality when it adopts the new product.

We demonstrate that, for an important range of demand conditions, the replacement and product inertia effects *exactly offset* when (1) the old product monopolist is protected from entry by another new product supplier, and (2) innovation is nondrastic in the sense that a joint monopolist supplies positive quantities of both old and new products. This implies that competition and protected monopoly provide *identical* incentives for innovation. When the old monopolist can be threatened by a new product entrant, monopoly provides *strictly greater* incentives. These findings stand in sharp contrast to the familiar result that competitive markets provide greater returns to process innovation.

Our analysis clarifies the conditions under which monopoly is socially preferable due to its effect on innovation. We first consider the case in which total R&D costs are not heavily affected by rivalry to discover the new product. Since a protected monopoly provides no greater incentive to innovate than does competition, social surplus under competition must be greater. A threatened monopoly may, however, bring forth innovation

that does not occur under competition, and threatened monopoly becomes superior when the value of innovation is great enough to outweigh the allocative efficiency of competition. These welfare rankings may be altered if total R&D spending is very sensitive to innovative rivalry, and in such cases product inertia may actually enhance welfare by slowing the pace of innovation, thereby reducing R&D costs. We illustrate the latter point below by means of a simple deterministic patent race model.

This paper contributes to a long-established tradition associated with Schumpeter, which emphasizes the importance of monopoly power for spurring innovation. Schumpeter argued that excessive competition in the market for the new product destroys innovation incentives, while our results show that competition in the old product market may be detrimental as well. Further, we develop ideas of Lange [1943] and Brozen [1951], who argue that product innovations may be discouraged relative to process innovations due to competition from old products.

More recent literature on patent rivalry (Dasgupta and Stiglitz [1980], Gilbert and Newbery [1982]) demonstrates that an established monopolist will have greater incentive to acquire a preemptive position in a new product than would an entrant, and therefore monopolies will tend to persist. This result and our conclusions arise from the same basic source: monopoly increases the returns from innovating by internalizing competitive externalities. The monopoly persistence result, however, hinges on the assumption that introducing the new technology preempts all rivals, while our basic conclusions continue to hold under various conditions of rivalry in the innovation activity and structure of new-product supply, as we show below.

Finally, this paper relates to the recent work on growth theory that emphasizes product improvements as the major source of growth, especially Stokey [1988, 1991], Aghion and Howitt [1990] and Grossman and Helpman [1991a,b]. In focusing on aggregate issues, these papers rely on very stylized specifications of market structure and interproduct

competition, and our work can be viewed as a first step in analyzing product improvements and growth in richer and more realistic structural settings.

The paper is organized as follows. Section 2 discusses several historical examples of intergenerational product competition, which serve to motivate our analysis. Section 3 outlines our basic model, and Section 4 compares the incentives to innovate under competitive and monopolistic structures of the old market. Section 5 compares the social surplus generated by competition and monopoly, and Section 6 extends our analysis to allow for rivalry in the innovative activity, i.e. patent racing, as well as oligopolistic rivalry in the new product market. Section 7 concludes the text. Proofs of propositions are given in Appendix 1, and Appendix 2 further develops the equality-of-returns result by showing that either protected monopoly or competition may provide greater returns for appropriate perturbations of the utility functions.

2. Examples of Intergenerational Product Competition

The historical record is replete with examples of innovations which, while being widely recognized as superior, displaced old technologies only gradually or incompletely. Consider first one of the most important product improvements of the late Nineteenth Century, the steel rail. Temin [1964] and Attack and Brueckner [1982] show that for the first two decades of its marketing, the principle use of Bessemer steel was for rails. Despite steel's superior durability, especially under heavy loads, iron rails continued to compete with steel. This was because steel rails were more costly until the late 1880's, and on less intensively used lines the extra durability of steel did not pay for its added cost. Iron rails were completely replaced only after the cost of steel finally dipped below that of iron.

The history of the mainframe computer is full of new inventions of systems with greater speed and internal memory. Each user would agree that the new systems offered superior performance, but not all desired the improvements enough to make up for their

added cost. Old and new generations of computer technology, both from the same firm and from competing firms, often simultaneously existed and competed for the same base of customers (Fisher, McKie and Mancke [1983]). As is well known, old generations were retired only after the costs of incorporating new capabilities had declined sufficiently.

Consumer markets provide many illustrations of intergenerational product competition. Consider the invention and improvement of household articles (Panati [1987]). Early aluminum cookware replaced cast iron pots, and later teflon-coated cookware competed against cheaper uncoated, although in each case the preceding generation has stayed in the market. Development of the electric blender has seen many minor (and sometimes costly) improvements, valued more by some consumers than by others, and not all immediately adopted - e.g. ice-crushing attachments, the coffee grinder head, and the capability of multiple speeds. Other examples of this kind of gradual innovation include the toaster, which evolved from a bread-baker to the automatic pop-up model, and the lawn mower, where manual rotary-blade mowers competed against more expensive motorized versions.

These examples illustrate two main points. First, since product innovations typically involve improvements that are not valued equally by all buyers, the postinnovation customer base tends to be segmented into those who value the improvements highly and buy the new product, and those who place less value on the improvements and stick with the old version. Competition between old and new products occurs at the margin between these two customer classes.

Second, while reductions in the cost of producing the new product might finally serve to drive out the old, the returns realized by the innovator may derive primarily from the initial phase of rivalry between old and new products. This could be due to discounting, to a slow pace of process improvements, or to difficulties an initial innovator may have in appropriating rents generated subsequent to the process improvements. In such cases, incentives to innovate are closely connected to the nature of intergenerational product

competition.

3. A Model of Intergenerational Product Competition

To capture these salient aspects of innovation, we propose the following model, which will form the foundation of our subsequent analysis. There is a basic good that comes in old and new versions. We assume that production technology for the old version exhibits constant returns to scale, with C_O denoting its per-unit production cost. Variable production costs for the new product are constant at C_N. In addition, the new product requires R&D costs, which we will consider in Sections 5 and 6.

Further, there is a continuum of consumers with heterogeneous tastes, indexed by $\omega \in [0,1]$, who are uniformly distributed on this interval with total mass one. Each consumer demands either zero units or one unit of the basic good, and either the old or the new product can be chosen. Let P_O and P_N be the prices of the old and new products. The net utilities for consumer ω purchasing the old and new products are given by $f_O(\omega) - P_O$ and $vf_N(\omega) - P_N$, where $v > 0$. The utility of not purchasing is zero.[2]

We assume that $f_O(\omega)$ and $f_N(\omega)$ are twice continuously differentiable, and that $f_O(0) = f_N(0) = 0$. In addition we suppose $f_O'(\omega)$, $f_N'(\omega) > 0$ for all ω, so that higher ω indicates stronger preference for the basic good, and that $f_O(1) > C_O$, so that the old product is viable. Define $\underline{v} > 0$ by:

$$\underline{v}f_N(1) - C_N = f_O(1) - C_O$$

For $v < \underline{v}$ the new product is not a viable competitor against the old, so we henceforth require $v \geq \underline{v}$. Assume further that $f_O'(\omega) < \underline{v}f_N'(\omega)$ for all ω; this implies that consumers $\omega > 0$ are

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2 Similar specifications of vertically differentiated product markets have been considered by Gabszewicz and Thisse [1979,1980] and Shaked and Sutton [1982,1983].

willing to pay a greater amount for the new product, and also that the premium is greater for consumers with higher ω. For technical purposes, we assume $\underline{v}f_N''(\omega) \leq f_O''(\omega) \leq 0$ for all ω (e.g., as a sufficient condition for concavity of profit functions). Finally, we impose the condition $f_N'(0) \leq C_O$, which is motivated below.

Figure 1 illustrates these demand conditions for prices $P_N > P_O > 0$. As shown, consumers with $\omega_N < \omega \leq 1$ maximize utility by purchasing the new product, those with $\omega_O < \omega < \omega_N$ purchase the old product, and those with $0 \leq \omega < \omega_O$ choose to make no purchase. The marginal consumers ω_O and ω_N are determined by:

$$f_O(\omega_O) - P_O = 0$$
$$f_O(\omega_N) - P_O = vf_N(\omega_N) - P_N$$

Manipulating these equations and setting $Q_O \equiv \omega_N - \omega_O$ and $Q_N \equiv 1 - \omega_N$ gives the inverse demand functions:

$$P_O = f_O(1 - Q_O - Q_N) \tag{1a}$$
$$P_N = vf_N(1 - Q_N) - f_O(1 - Q_N) + f_O(1 - Q_O - Q_N) \tag{1b}$$

Our analysis will focus chiefly on three types of postinnovation market structures. In each case the new product is taken to be monopolized, e.g. by a patent-holding inventor, but the structure of the old product market is varied. The cases are: (1) *joint monopoly*, in which a single firm monopolizes both the old and new products; (2) *dominant-fringe structure*, in which the old product is supplied competitively; and (3) *differentiated duopoly*, in which the old product is monopolized by a firm distinct from the new product monopolist. In the dominant-fringe case, we assume that the dominant firm has a first-mover advantage in choosing quantities, while for differentiated duopoly we presume Nash equilibrium in

quantity choices. Postinnovation market structures in which the new product need not be monopolized will be considered in Section 6b.

We will say that the new product represents a *drastic innovation* if the presence of the old product has no effect on the maximized profits of the new product monopolist, i.e. the latter may simply act as if the old product did not exist. This notion is analogous to Bain's [1949] concept of blockaded entry, where the old product suppliers are thought of as entrant firms. If the presence of the old product constrains the new product monopolist, then innovation is *nondrastic*.[3] Regions of drastic and nondrastic innovation under the three market structures are characterized in the following proposition, whose proof is given in Appendix 1:

Proposition 1. There exist v^A and v^B, satisfying $\underline{v} < v^A < v^B$, such that (i) under joint monopoly, innovation is nondrastic if $\underline{v} \leq v < v^A$, and drastic if $v \geq v^A$; and (ii) under dominant-fringe structure and differentiated duopoly, innovation is nondrastic if $\underline{v} \leq v < v^B$, and drastic if $v \geq v^B$.

Observe that under joint monopoly, innovation is nondrastic for a strictly smaller range of v than under dominant-fringe and differentiated duopoly structures. In proving the proposition, we invoke the condition $f_N'(0) \leq C_0$ to rule out the possibility that innovation is nondrastic for *all* v.[4]

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[3] Arrow [1962] defines a process innovation to be drastic if and only if the monopoly price under the new technology lies below the competitive price under the old technology; hence the profits of a monopolist operating the new technology are unaffected by the existence of the old. Our notion of drastic innovation is an extension of this basic idea to the case of product innovation.

[4] To see how the latter may happen, consider the case of joint monopoly and suppose the monopolist chooses $Q_O = 0$. The elasticity of demand for the new product at $Q_O = 0$ is, using (1b), $\varepsilon(Q_N) = f_N(1 - Q_N)/f_N'(1 - Q_N)Q_N$. Note that demand elasticity is independent of v and strictly decreasing in Q_N. Profit maximizing choice of Q_N given $Q_O = 0$ requires $\varepsilon(Q_N) > 1$.

4. Returns to Innovation

We now turn to the central question of our paper: what market structure generates the greatest returns to innovation? More specifically, how does the structure of intergenerational product competition impact on innovation incentives? To clarify matters it is convenient to frame the question in terms of Arrow's [1962] conceptual experiment, in which the innovation is controlled by a patent-holding inventor, whose returns are generated by licensing revenue. Thus the question becomes, what market structure allows the inventor to earn the greatest possible revenue?

One can distinguish two channels by which innovation is affected by intergenerational product competition: first, on the demand side, the old and new products are imperfect substitutes; and second, on the supply side, various structural links between old and new products may limit firms' abilities to market the new product. These factors lead us to consider four possible preinnovation market situations. This first is (1) *competition*, in which the old product is competitively supplied and any firm may produce the new product. In this case the products are linked only by the structure of demand. The remaining cases, which posit monopoly in the old product market, are: (2) *protected monopoly*, in which the old product monopolist is the only firm that is capable of producing the new product, e.g. because of specialized factors of production controlled by the monopolist; (3) *threatened monopoly*, in which any firm, including the old product monopolist and new entrants, may adopt the new product; and (4) *excluded monopoly*, in which only new entrants may produce

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Further, let Q_N' denote the smallest level of Q_N such that the marginal profitability of the old product at $Q_O = 0$ is nonpositive. As long as $\varepsilon(Q_N') \leq 1$, it is never be optimal for the monopolist to choose $Q_O = 0$, no matter how large v is. The condition $f_N'(0) \leq C_O$ implies $\varepsilon_N(Q_N') > 1$, however, and innovation becomes drastic once v reaches the critical level v^A. Similar comments apply with respect to the other market structures.

the new product, e.g. due to legal restrictions.

Consider first the competitive case. Here the inventor may license to as many firms as desired, and he can appropriate any profits generated by the new product by means of licensing fees. Denote by Π^C the profits earned by the new product monopolist under dominant-fringe structure; thus Π^C gives the maximum licensing revenue of the inventor, which the inventor may obtain by licensing to a single firm that becomes the new product monopolist.

The situation is different under protected monopoly, in that the inventor can license only to the old product monopolist. By adopting the new product the latter earns the joint monopoly profits, denoted Π^M, less the licensing fee, and adoption will be chosen only if these postadoption profits exceed what the monopolist would have earned by simply keeping its old product monopoly; profits in the latter case are written Π^{OM}. Thus, $\Pi^M - \Pi^{OM}$ gives the maximum licensing revenue available to the inventor under protected monopoly.

Competitive market structure provides greater returns to the inventor than does protected monopoly if $\Pi^C > \Pi^M - \Pi^{OM}$, which can be reexpressed as:

$$\Pi^{OM} > \Pi^M - \Pi^C$$

On the left-hand side we have the difference between firms' opportunity costs of adoption under protected monopoly and competition, which is certain to be strictly positive. This is the *replacement effect* discussed by Arrow, and it may be seen that the effect tends toward giving competition the advantage as far as innovation incentives.

On the right-hand side we have the difference between the total benefits of adoption under competition and monopoly, and this will be strictly positive as long as the new product does not completely displace the old. This is a new effect: competition from the old product, which takes the form of a competitive fringe of old product suppliers, limits the

rents that can be extracted from the new product. The old product acts as a drag on the incentives to adopt the new; hence, we call this the *product inertia effect.* Note that this effect tends toward providing greater incentives under monopoly.

Since the replacement and product inertia effects cut in opposite directions, it is not immediately clear which market structure provides greater incentives for innovation. In fact, we can show that when protected monopoly leads to nondrastic innovation, i.e. when a joint monopolist supplies positive quantities of both old and new products, the two effects *exactly offset*, and protected monopoly and competition provide precisely the same returns to innovation. This remarkable result hinges on the relationships between pre- and postinnovation quantities. Let Q_O^M and Q_N^M denote the quantities supplied under joint monopoly, Q_N^C the new product supply under dominant-fringe structure, and Q_O^{OM} the monopoly supply of the old product when the new product has not been adopted. In Appendix 1 we prove the following:

Lemma. If $\underline{v} \leq v \leq v^A$, then:

(a) $Q_O^{OM} = Q_O^M + Q_N^M$, i.e. the protected monopolist's total output is the same whether or not it adopts the new product; and

(b) $Q_N^M = Q_N^C$, i.e. output of the new product under joint monopoly is identical to output under dominant-fringe structure.

The Lemma places important restrictions on the equilibrium quantities when innovation is nondrastic under joint monopoly. To develop some feel for these restrictions, let us express the profit function of the joint monopolist in terms of the total output $Q \equiv Q_O + Q_N$ and the new product output Q_N; using (1a) and (1b), we have:

$$(2) \qquad [f_O(1 - Q) - C_O][Q - Q_N] + [vf_N(1 - Q_N) - f_O(1 - Q_N) + f_O(1 - Q) - C_N]Q_N$$

$$= [f_O(1 - Q) - C_O]Q + [vf_N(1 - Q_N) - f_O(1 - Q_N) + C_O - C_N]Q_N$$

Observe that on the right-hand side of (2), profits have been decomposed into two components, with one component depending only on Q and the other depending only on Q_N. Part (a) follows from the fact that the Q component has exactly the same form as the profit function of a nonadopting monopolist. Essentially, the total quantity produced in the nondrastic case hinges on the marginal old product purchaser, and under our demand structure this purchaser's behavior is the same whether or not the monopolist adopts the new product.

Part (b) is a bit more subtle. Note first that under dominant-fringe structure, the competitive fringe imposes an "implicit tax" on the new product supplier; using (1a), (1b) and $P_O = C_O$, we may express the new-product price as:

$$(3) \qquad P_N = vf_N(1 - Q_N) - \{f_O(1 - Q_N) - C_O\}$$

The implicit tax is given by the term in braces. Similarly, the existence of the old product reduces the new product price under joint monopoly, but the effect is more complex since the monopolist earns profits from both products. Note however that in the Q_N component of the right-hand side of (2), the effective new product price is exactly the same as (3). Since the implicit tax is the same in each case, the incentives to produce the new product are also the same.

The equality of implicit tax rates is explained by two effects. First, the existence of the old product under joint monopoly directly reduces P_N by an amount $f_O(1 - Q_N) - P_O$, as can be seen in (1b). When innovation is nondrastic, however, the joint monopolist reduces

its sales of the old product at a one-to-one rate when it sells additional units of the new; this imposes an added tax of $P_O - C_O$, which is the profit margin on the old product. The sum of these effects yields an implicit tax rate that is identical to the level under dominant-fringe structure.

We now establish $\Pi^C = \Pi^M - \Pi^{OM}$. Using the Lemma and (2), maximized profits under joint monopoly may be written:

$$\Pi^M = [f(1 - Q^{OM}) - C_O]Q^{OM} + [vf_N(1 - Q_N^C) - f_O(1 - Q_N^C) + C_O - C_N]Q_N^C$$

$$= \Pi^{OM} + \Pi^C$$

which establishes the result. In essence, the implicit tax rates under dominant-fringe and joint monopoly structures serve as per-unit measures of the product inertia and replacement effects, respectively. Equality of the tax rates directly reflects equality of the two effects.

For innovations of greater value, in particular those that are drastic under joint monopoly but not under dominant-fringe structure, the product inertia effect is partially attenuated by the fact that $Q_N^C > Q_M^N$ when $v^A < v < v^B$. As a consequence, competition provides strictly greater returns than does protected monopoly. The arguments supporting this result are given in Appendix 1, and here we summarize with:

Proposition 2. (a) If $\underline{v} \leq v \leq v^A$, then $\Pi^C = \Pi^M - \Pi^{OM}$, and competition and protected monopoly provide the same returns to innovation;
(b) If $v > v^A$, then $\Pi^C > \Pi^M - \Pi^{OM}$, and competition provides strictly greater returns.

The equality of returns shown in part (a) of Proposition 2 establishes that competition and protected monopoly can provide equal incentives to innovate under reasonable market

conditions. It is appropriate to ask, however, whether the equality is part of a larger weak inequality over a broader class of demand conditions, or whether equality itself holds up for broader conditions. This question is addressed in Appendix 2, where it is shown that by appropriately perturbing the utility functions in the $\underline{v} < v \leq v^A$ case, either competition or protected monopoly may provide strictly greater returns; thus equality should be interpreted to mean that returns will be very close over a large range of demand conditions, but that it is possible for either market structure to provide greater returns.

Now consider the case of threatened monopoly, in which the inventor can license the innovation to any firm. As with competition, the inventor does best by licensing to whatever number of firms maximizes industry profits, and the latter is accomplished by selling the rights to a single firm. If this firm is a new entrant, then by adopting the innovation it earns the new product monopolist's Nash equilibrium profit, given by Π^{ND}, less the licensing fee, as adoption leads to a differentiated duopoly that pits the entrant against the old product monopolist. The opportunity cost of adoption for the new entrant is the normal profit rate, i.e. zero.

If the inventor sells to the old product monopolist, then the latter's profits upon adoption are Π^M less the licensing fee. The monopolist's opportunity cost is the Nash equilibrium profit of the old product monopolist, written Π^{OD}, since the inventor will sell to a new entrant if the monopolist declines to adopt. Thus $\Pi^M - \Pi^{OD}$ is the maximum revenue that the inventor can gain by licensing to the old product monopolist. Now, as long as $\underline{v} \leq v < v^B$ we have $\Pi^M > \Pi^{OD} + \Pi^{ND}$, i.e. joint monopoly yields strictly greater industry profits than does differentiated duopoly, and it is apparent that the inventor's best policy is to license to the old product monopolist (this is an instance of Gilbert and Newbery's [1982] "persistence of monopoly" result).

In view of Proposition 2, it is simple to compare competition and threatened monopoly for the nondrastic case: we have $\Pi^{OD} < \Pi^{OM}$, since the entrant cuts into the old

product monopolist's market, and thus $\Pi^{M} - \Pi^{OD} > \Pi^{M} - \Pi^{OM} = \Pi^{C}$ when innovation is nondrastic under joint monopoly. In this case, the replacement effect is mitigated by the threat of entry. The resulting dominance of the product inertia effect means that threatened monopoly provides *strictly greater* returns to innovation than does competition. In Appendix 1 we extend this conclusion to the interval (v^{A},v^{B}), on which innovation is nondrastic under dominant-fringe structure but not under joint monopoly. This establishes:

Proposition 3. (a) If $\underline{v} \leq v < v^{B}$, then $\Pi^{M} - \Pi^{OD} > \Pi^{C}$, and threatened monopoly provides strictly greater returns to innovation than does competition;

(b) If $v \geq v^{B}$, then $\Pi^{M} - \Pi^{OD} = \Pi^{C}$, and the two market structures provide the same returns.

Finally we consider excluded monopoly. Since the inventor cannot license to the old product monopolist, his maximum licensing revenue becomes Π^{ND}, and since $\Pi^{ND} < \Pi^{M} - \Pi^{OD}$ for $\underline{v} \leq v < v^{B}$ it follows that excluded monopoly provides strictly lower returns to innovation than does threatened monopoly. In Appendix 1 we demonstrate $\Pi^{C} < \Pi^{ND}$ for these levels of v, and this gives:

Proposition 4. (a) If $\underline{v} \leq v < v^{B}$, then $\Pi^{M} - \Pi^{OD} > \Pi^{ND} > \Pi^{C}$, and excluded monopoly provides returns to innovation that are strictly between those of threatened monopoly and competition;

(b) If $v \geq v^{B}$, then $\Pi^{M} - \Pi^{OD} = \Pi^{ND} = \Pi^{C}$, and all of the market structures provide the same returns.

The superiority of excluded monopoly over competition in the nondrastic case illustrates how the product inertia effect is sensitive to the intensity of competition from the

old product market. Differentiated duopoly conveys greater market power in the new-product market than does dominant-fringe structure, and as a consequence innovation under excluded monopoly generates more rents for the inventor to extract. This suggests a more general conclusion: if innovation is nondrastic, then returns to innovation rise as the old product market becomes more concentrated.[5] In this way, product inertia implies a direct link between monopoly power and innovation incentives, which is distinctly different from the link that arises from the replacement effect.

5. Social Welfare

In this section we consider normative aspects of the structure of intergenerational product competition. While competition in the old product market yields better allocative efficiency for a given set of products, monopoly may give rise to innovation that would not occur under competition. Monopoly may then be superior on balance if the innovation is of sufficient value. Let S^C_{NA} denote the social surplus obtaining from competition in the old product market when the new product is not adopted, and let S^M_A denote the surplus associated with an old product monopoly that adopts the new product. This tradeoff between market structures is made explicit in the following proposition, which is proved in Appendix 1:

Proposition 5. There exists v^*, satisfying $\underline{v} < v^* < v^B$, such that $S^C_{NA} > S^M_A$ if $v < v^*$, $S^C_{NA} = S^M_A$ if $v = v^*$, and $S^C_{NA} < S^M_A$ if $v > v^*$.

According to this proposition, there is always a level v^* such that for v above this level, monopoly with adoption provides strictly greater social surplus that competition without adoption. It follows that normative comparison of the market structures depends on

[5] We will examine this conjecture in greater detail in a forthcoming paper.

the incentives that are provided to adopt the new product, together with the value of the innovation. Let us consider the very simple situation in which the inventor must incur a fixed R&D cost of $F > 0$ in order to bring forth the new product. This can be thought of as a limiting case in which rivalry to acquire the patent does not affect the realized level of R&D expenditures; we defer to Section 6a the more detailed case of patent races between rival potential inventors. In the present instance, adoption of the new product occurs if and only if the returns from adoption exceed F. It is clear from Proposition 2 that competition provides strictly greater social welfare than protected monopoly, since $\Pi^C \geq \Pi^M - \Pi^{OM}$ implies that adoption occurs under competition whenever it occurs under protected monopoly. Threatened monopoly may provide greater welfare than competition, however, since we have $\Pi^M - \Pi^{OD} > \Pi^C$ for $\underline{v} < v < v^B$.

The latter comparison is illustrated in Figure 2, which depicts $\Pi^M - \Pi^{OD}$ and Π^C as functions of v for a given specification of the functions $f_O(\omega)$ and $f_N(\omega)$.[6] For $F \leq \Pi^C$, adoption occurs under both threatened monopoly and competition, while for $F > \Pi^M - \Pi^{OD}$ adoption occurs under neither; in these regions competition is superior based on allocative efficiency. In the area between the curves, labelled Regions 1, 2 and 3, adoption occurs under threatened monopoly but not under competition. In Regions 1 and 2 we have $S_A^M - F < S_{NA}^C$, so competition remains superior. In Region 3, however, the value of innovation is great enough to outweigh the efficiency advantage of competition, and the inequality is reversed. In this region it is threatened monopoly that provides the greater social welfare.

Interestingly, in Region 1 social welfare under monopoly would be greater if the monopolist did not adopt the innovation (S_{NA}^M denotes surplus under monopoly in the old

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[6] In particular, we specify $f_O(\omega) = f_N(\omega) = \omega$. Figure 2 is calculated for the values $C_O = 4/7$ and $C_N = 6/7$; this gives $\underline{v} = 9/7$, $v^A = 1.5$ and $v^B = 6$. The propositions hold for this example despite the violation of the assumption $f_N'(0) \leq C_O$; for $f_O(\omega) = f_N(\omega) = \omega$ the assumption may be replaced by $C_O > 1/2$.

product market when the new product is not adopted). This exemplifies the familiar proposition that rivalry may lead to socially excessive innovation, where in this instance it is product market rivalry, rather than rivalry to acquire the patent, that leads to excessive adoption. Finally, excluded monopoly and competition may be compared by noting that the curve Π^{ND} lies between Π^{M} - Π^{OD} and Π^{C}, so that the areas corresponding to Regions 1, 2 and 3 become smaller.

6. Extensions

a. Rivalry in the Innovative Activity

In the preceding analysis we have assumed the point of view of an inventor whose innovation decision is essentially static, and who faces no rivals in the innovative activity. Actual R&D activity, however, has important dynamic aspects and frequently involves intense rivalry among potential discoverers of the innovation. Further, rivalry typically affects the amount of R&D expenditures that are undertaken, and thereby exerts an added effect on net social surplus. In this section we extend our analysis to allow the timing of innovation to be determined by a simple deterministic patent race. The main new finding is that in situations where delaying innovation generates large reductions in R&D costs, product inertia may become favorable for welfare, due to its tendency to slow innovation and thereby to mitigate against excessive R&D expenditures.

We consider a patent race model similar to that of Gilbert and Newbery [1982]. There is a pool of potential innovators, any of whom can discover the innovation at time T by paying an R&D cost of F(T). Time is continuous, and F(T) is positive and strictly decreasing in T; the assumption of declining R&D costs can be justified in terms of complementary discoveries in other sectors, or diseconomies stemming from compression of research activity. When one innovator makes the discovery at T, he obtains a patent on the product, and his subsequent profits are determined as above. We analyze the natural analog

of subgame-perfect equilibria for this continuous-time setting.

Suppose first that the pool of potential innovators includes any agent that desires to make the R&D investment, so that R&D activity is disintegrated from the production process for the new product; we call this *disintegrated R&D*. Innovation occurs as soon as R&D costs exactly dissipate all rents available from the patent. In the case of competition, for example, the equilibrium discovery time is determined by:

$$\frac{\Pi^C}{r} - F(T) = 0 \tag{4}$$

where r is the rate of discount, and we have assumed for simplicity that the flow of profits from the new product continues unchanged for all time. The discovery time is determined similarly for protected and threatened monopoly.

Let us now consider social surplus in patent race equilibria for the specification of Figure 2, where we also set $F(T) = F(0)e^{-dT}$. The parameter $d > 0$ indicates the rate at which R&D costs decline over time. Further, we put $F(0) = \Pi^M - \Pi^{OD}$, i.e. time zero is taken to be the instant at which adoption occurs under threatened monopoly; measuring discounted social surplus at $T = 0$ then serves to maximize the relative advantage of threatened monopoly versus competition.[7]

The results are summarized in Figure 3. For $v < v^B$, adoption occurs sooner under threatened monopoly than under competition. But threatened monopoly gives greater equilibrium social surplus than competition only on Region 1, where d is small and the value of innovation is relatively high. The small d case approximates the static situation of Section

[7] Measuring social surplus at other times will increase the weight placed on times at which adoption has occurred under neither or both threatened monopoly and competition, and at such times competition is more attractive. Thus our results give an upper bound on the attractiveness of threatened monopoly. Issues of timing and social welfare are considered further in DeBrock and Masson [1985].

5; in particular, as d approaches negative infinity, the patent race outcomes converge to the points on the curve $\Pi^M - \Pi^{OD}$ in Figure 2, and it follows from above that threatened monopoly is superior if v is sufficiently large. For larger d, delayed adoption leads to greater marginal reductions in R&D costs, and as a consequence the slower adoption makes competition more attractive. Here product inertia becomes desireable precisely because it slows innovation.

A similar effect arises when competition and protected monopoly are compared. Adoption occurs no later under competition, but in Region 4 social surplus is greater under protected monopoly due to the savings in R&D costs resulting from slower adoption. Now it is the replacement effect that becomes relatively favorable for welfare. Interestingly, the relative advantage of protected monopoly increases as v rises, since an increase in v serves to widen the gap between adoption times. Finally, threatened monopoly is superior to protected monopoly only on Regions 1 and 2, where again the slower adoption under protected monopoly becomes attractive when d is large.

We have assumed thus far that R&D activities and production of the new product may be carried out by distinct agents. Let us now consider the alternative possibility that R&D is directly related to the production process, so that the producing firm must pay its own R&D cost rather than licensing from an outsider; this is called *integrated R&D*. We assume as before that the first firm to pay the R&D cost wins a patent over the new product, and that profits from the new product continue unchanged for all time.

The distinction between integrated and disintegrated R&D is immaterial under competition, as any firm can produce the new product; thus (4) continues to give the equilibrium discovery time. Protected monopoly is affected, however, since integrated R&D gives the old-product producer a monopoly over the innovative activity. Now the firm chooses its discovery time to maximize profits net of R&D costs, and the first-order condition for this maximization is:

$$\frac{\Pi^{M} - \Pi^{OM}}{r} - F(T) + \frac{F'(T)}{r} = 0$$

Since $F' < 0$, it follows that discovery occurs strictly later when R&D is integrated. Threatened monopoly is similarly affected, except that new entrants can earn Π^{ND} by innovating, and this places an upper bound on how long the threatened monopolist can delay innovation.

Figure 4 summarizes equilibrium social surplus in the integrated R&D case, for the same specification of the model as in Figure 3. For low values of d, the reductions in R&D costs from delay are small relative to the surplus that consumers earn from the new product, so that delay is excessive under protected and threatened monopoly. Thus Region 1, on which threatened monopoly is superior to competition, is smaller here than in the disintegrated R&D case of Figure 3. For large d, in contrast, delay leads to large cost reductions, and protected and threatened monopoly become relatively more attractive; note that Region 4, on which protected monopoly dominates competition, becomes much larger under integrated R&D due to the added delay under protected monopoly.

b. Rivalry in the New Product Market

Thus far we have assumed that the new product is controlled exclusively by a patent-holding inventor. In many cases, however, patent protection is not available to developers of new products, and there arises rivalry between firms that market different variants of the new technology. In this section we show that product inertia continues to play a fundamental role when there is the potential for rivalry in the new product market.

We now suppose that two firms produce the old product. Each has the option of adopting the new product at a cost of F. We consider a two-stage game between these firms:

In the first stage the firms choose simultaneously whether or not to market the new product. In the second stage, the firms choose quantities to produce, where the new product can be produced only if the firm had decided to market it in the first stage, while the old product can be produced irrespective of the stage one decision. Demand and costs take the same form as above. We study subgame-perfect equilibria of this game, in which the quantity decisions give Nash equilibria of the second stage for every possible adoption profile in the first stage.[8]

Figure 5 summarizes the pure-strategy adoption equilibria in terms of of F and v, for the specification considered in Figure 2.[9] In Regions 1a-d, neither firm adopts due to the high adoption costs. In Region 2 only one firm adopts, and in Regions 3a-b both firms adopt. Under threatened monopoly, in contrast, there is no adoption in Regions 1a and 1b, while adoption occurs in the remaining regions. Thus Region 1c and 1d are associated with outcomes in which adoption fails to occur in the duopoly case, even though a threatened monopolist would adopt. Here product inertia derives from the rival producer of the old product: although either firm could adopt and acquire a monopoly position in the new product market, the presence of the rival reduces the returns to adoption and makes it unattractive relative to threatened monopoly.

Let us now compare the adoption equilibria to the adoption profiles that would maximize social surplus. Social surplus is calculated under the assumption that outputs are determined by Cournot quantity-setting, i.e. imperfect competition in the quantity stage is

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[8] R&D rivalry in a differentiated duopoly has been considered in a number of recent papers: Shaked and Sutton [1990] analyze an adoption game similar to ours for abstract reduced-form profit functions; Ramey [1988] and Bagwell and Staiger [1990] consider models with random R&D outcomes; and Beath, Katsoulacos and Ulph [1987], Scotchmer and Green [1990], Dutta, Lach and Rustichini [1990] and Aoki [1991] study the nature of leader-follower relationships when a sequence of product improvements can be adopted.

[9] Duopoly equilibria are unique on the interiors of Regions 1 and 3, in which neither adopt or both adopt. In Region 2, there exist a pair of asymmetric pure-strategy equilibria in which only one firm adopts, as well as a single symmetric mixed-strategy equilibrium.

taken as a constraint. In Regions 1a and 1c of Figure 5, nonadoption maximizes social surplus, while in Regions 1b, 1d, 2 and 3a the highest social surplus is associated with adoption by only one firm. Adoption by both firms is optimal only in Region 3b. It follows that adoption is suboptimally low in Regions 1b and 1d, as a consequence of product inertia together with incomplete appropriability of consumer surplus, while adoption is suboptimally high in Region 3a. In the latter region, which is associated with relatively high v for given F, most of the gains in consumer surplus are generated by the initial adoption, and the cost of the second adoption outweighs the gain in surplus from having lower concentration in the new-product market.

7. Conclusion

Our analysis demonstrates that for a widely observed pattern of product innovation, in which old and new products compete for a common customer base, monopoly may provide greater incentives for innovation than does competition. The key new effect is product inertia, wherein competition from the old product reduces the rents available from the new. When innovation is nondrastic under monopoly, our model demonstrates that the incentives to innovate under competition and protected monopoly are identical. Further, incentives are strictly greater under monopoly when the monopolist is threatened. Threatened monopoly may provide greater social welfare than competition when the innovation is sufficiently valuable to offset the allocative efficiency of competition, and realized R&D costs are not excessively sensitive to rivalry in the innovative activity.

Stated more briefly, monopoly threatened with entry gives greater social welfare when innovations are valuable and adoption costs are high, while competition is best when adoption costs are low. This welfare analysis suggests a classification of industries by scope for innovation. Highly innovative sectors in which entirely new markets emerge from rapid technological progress (computers, communication equipment, biotech) may provide greater

welfare when markets are monopolized. In sectors where new technologies are less fundamental and innovation takes the form of nonprice competition via small product improvements (food and household items, apparel), welfare is higher under competition, since the gains from innovation are small. As pointed out above, this classification may be reversed if excessive R&D expenditures become the dominant consideration.

In future work we plan to explore in more detail the link between concentration in old and new product markets and R&D incentives. Further, in this paper we have taken the degree of product improvement to be exogeneous, and it would be useful to assess the effect of market structure on firms' product improvement decisions, as well as to incorporate spillovers in product design into the analysis. Cost-reducing, quality-reducing innovation can also be considered within our basic framework. Overall, we feel that explicit consideration of intergenerational product competition will continue to provide rich insights into the nature of innovation incentives.

Appendix 1

Proofs of Propositions and Lemma

The proof of Proposition 1 is given in the following three lemmata.

Lemma A1. There exists $v^A > \underline{v}$ defined jointly with Q_N^M by:

(A1a) $$-f_O'(1 - Q_N^M)\, Q_N^M + f_O(1 - Q_N^M) - C_O = 0$$

(A1b) $$-v^A f_N'(1 - Q_N^M)\, Q_N^M + v^A f_N(1 - Q_N^M) - C_N = 0$$

such that innovation is nondrastic under joint monopoly when $\underline{v} < v < v^A$, and drastic when $v \geq v^A$.

Proof. The profits of a joint monopolist may be written, using (1a) and (1b):

(A2) $$\Pi = [f_O(1 - Q_O - Q_N) - C_O]\, Q_O + [v f_N(1 - Q_N) - f_O(1 - Q_O) + f_O(1 - Q_O - Q_N) - C_N]\, Q_N$$

Under our assumptions (A2) is a strictly concave function; let Q_O^M and Q_N^M denote the unique maximizers of (A2). We have $Q_N^M > 0$ as a consequence of $v > \underline{v}$. The remaining possibilities are $Q_O^M > 0$, which constitutes nondrastic innovation, and $Q_O^M = 0$, which is drastic innovation. Q_O^M and Q_N^M in the nondrastic case are characterized by:

(A3a) $$-f_O'(1 - Q_O^M - Q_N^M)\, [Q_O^M + Q_N^M] + f_O(1 - Q_O^M - Q_N^M) - C_O = 0$$

(A3b) $$-f_O'(1 - Q_O^M - Q_N^M)\,[Q_O^M + Q_N^M] - [vf_N'(1 - Q_N^M) - f_O'(1 - Q_N^M)]\,Q_N^M + vf_N(1 - Q_N^M) - f_O(1 - Q_N^M) + f_O(1 - Q_O^M - Q_N^M) - C_N = 0$$

and in the drastic case by:

(A4a) $$-f_O'(1 - Q_N^M)\,Q_N^M + f_O(1 - Q_N^M) - C_O \le 0$$

(A4b) $$-vf_N'(1 - Q_N^M)\,Q_N^M + vf_N(1 - Q_N^M) - C_N = 0$$

Let Q_N' be the level of Q_N that gives equality in (A4a). Since $Q_N' < 1$ necessarily, we may choose v sufficiently close to $\underline{v}$ to give:

$$vf_N(1 - Q_N') - C_N - f_O(1 - Q_N') + C_O < 0$$

If $Q_N^M \ge Q_N'$, then:

$$\begin{aligned} &vf_N(1 - Q_N^M) - C_N - f_O(1 - Q_N^M) + C_O \\ &= vf_N(1 - Q_N') - C_N - f_O(1 - Q_N') + C_O - \int_{1-Q_N^M}^{1-Q_N'} [vf_N'(\omega) - f_O'(\omega)]d\omega < 0 \end{aligned}$$

Thus if Q_N^M satisfied (A4b) we would have:

$$\begin{aligned} 0 &= -vf_N'(1 - Q_N^M)\,Q_N^M + vf_N(1 - Q_N^M) - C_N \\ &< -f_O'(1 - Q_N^M)\,Q_N^M + f_O(1 - Q_N^M) - C_O \end{aligned}$$

and (A4a) would be violated. It follows that innovation must be nondrastic for v sufficiently

close to $\underline{v}$.

Observe next that our assumptions imply, for all $\omega > 0$:

$$f_O(\omega) - f_N(\omega) - f_O'(\omega)\,[1 - \omega] + f_N'(\omega)\,[1 - \omega]$$
$$< f_N'(0) \leq C_O$$

Rearranging gives:

$$-f_O'(\omega)\,[1 - \omega] + f_O(\omega) - C_O < -f_N'(\omega)\,[1 - \omega] + f_N(\omega)$$

and at $\omega = 1 - Q_N'$ we have:

$$0 = -f_O'(1 - Q_N')\,Q_N' + f_O(1 - Q_N') - C_O < -f_N'(1 - Q_N')\,Q_N' + f_N(1 - Q_N')$$

It follows that for sufficiently large v:

$$-vf_N'(1 - Q_N')\,Q_N' + vf_N(1 - Q_N') - C_N > 0$$

and (A4b) will be satisfied by $Q_N^M > Q_N'$, which implies satisfaction of (A4a). Thus innovation is drastic for sufficiently large v.

Note finally that the Q_N^M satisfying (A4b) is strictly increasing in v, which implies that once innovation is drastic at a given v, it continues to be drastic for all larger v. From (A1a) and (A1b) we have that innovation is nondrastic when $v < v^A$ and drastic when $v \geq v^A$. Q.E.D.

Lemma A2. There exists $v^B > v^A$ defined jointly with Q_N^L by:

(A5a) $$f_O(1 - Q_N^L) - C_O = 0$$

(A5b) $$-v^B f_N'(1 - Q_N^L)\, Q_N^L + v^B f_N(1 - Q_N^L) - C_N = 0$$

such that innovation is nondrastic under dominant-fringe structure when $\underline{v} < v < v^B$, and drastic when $v \geq v^B$

Proof. Let Q_O^C denote the fringe supply. If the new product monopolist chooses $Q_N < Q_N^L$, where Q_N^L is defined by (A5a), then Q_O^C is determined by the requirement that price equal marginal cost in the old product market:

$$f_O(1 - Q_O^C - Q_N) - C_O = 0$$

while if $Q_N \geq Q_N^L$ we have $Q_O^C = 0$; thus Q_N^L is the limit quantity. If $Q_N \leq Q_N^L$, then the monopolist's profits are:

$$\Pi = [vf_N(1 - Q_N) - f_O(1 - Q_N) + C_O - C_N]\, Q_N$$

which is a strictly concave function. Let $\underline{Q}_N^C$ denote its maximizer on $[0, Q_N^L]$; we have $\underline{Q}_N^C > 0$ as a consequence of $v > \underline{v}$. If $\underline{Q}_N^C < Q_N^L$, then $\underline{Q}_N^C$ is characterized by:

(A6) $$\begin{aligned} &-[vf_N'(1 - \underline{Q}_N^C) - f_O'(1 - \underline{Q}_N^C)]\, \underline{Q}_N^C \\ &+ vf_N(1 - \underline{Q}_N^C) - f_O(1 - \underline{Q}_N^C) + C_O - C_N = 0 \end{aligned}$$

If instead we have $Q_N \geq Q_N^L$, then the monopolist's profit function is:

$$\Pi = [vf_N(1 - Q_N) - C_N]\, Q_N$$

which again is strictly concave. Let $\dot{Q}_N^C$ denote the maximizer of this function over <u>all</u> quantities, including $Q_N < Q_N^L$; thus $\dot{Q}_N^C$ is defined by (A4b), with $\dot{Q}_N^C$ replacing Q_N^M. It follows that innovation is drastic as long as $\dot{Q}_N^C \geq Q_N^L$, since the new product monopolist may simply implement the unconstrained profit maximum by choosing $Q_N^C = \dot{Q}_N^C$, while if $\dot{Q}_N^C <$ Q_N^L the presence of the competitive fringe affects the monopolist's profits, so that innovation is nondrastic. In the latter case the monopolist's optimal choice is $Q_N^C = \underline{Q}_N^C$. Note that we may have $Q_O^C = 0$ when innovation is nondrastic, if the monopolist elects to pursue a "limit pricing" strategy by choosing $\underline{Q}_N^C = Q_N^L$.

The argument of Lemma A1 is easily modified to establish that innovation is nondrastic for v sufficiently close to $\underline{v}$ (replace Q_N' with Q_N^L). Further, the condition $f_N'(0) \leq$ C_0 implies:

(A7)
$$-f_N'(1 - Q_N^L)\, Q_N^L + f_N(1 - Q_N^L) > 0$$

so that $\dot{Q}_N^C > Q_N^L$ for sufficiently large v. Since $\dot{Q}_N^C$ is strictly increasing in v, it follows that a $v^B > \underline{v}$ exists having the desired property. Finally, at $v = v^A$ we have $Q_N^M = \dot{Q}_N^C$ and thus $\dot{Q}_N^C$ satisfies (A1a); this implies $\dot{Q}_N^C < Q_N^L$, so that we must have $v^B > v^A$. Q.E.D.

Lemma A3. Innovation is nondrastic under differentiated duopoly when $\underline{v} < v < v^B$, and drastic when $v \geq v^B$, where v^B is defined by (A5a)-(A5b).

Proof. Under differentiated duopoly the profit functions for the old and new monopolists respectively are:

$$\Pi_O = [f_O(1 - Q_O - Q_N) - C_O]\, Q_O$$

$$\Pi_N = [vf_N(1 - Q_N) - f_O(1 - Q_N) + f_O(1 - Q_O - Q_N) - C_N]\, Q_N$$

Let the Nash equilibrium quantities be denoted Q_O^D and Q_N^D. As in the preceding two lemmas, $v > \underline{v}$ assures $Q_N^D > 0$. Equilibria with $Q_O^D > 0$ are characterized by:

(A8a) $$-f_O'(1 - Q_O^D - Q_N^D)\, Q_O^D + f_O(1 - Q_O^D - Q_N^D) - C_O = 0$$

(A8b) $$\begin{aligned} &-[vf_N'(1 - Q_N^D) - f_O'(1 - Q_N^D) + f_O'(1 - Q_O^D - Q_N^D)]\, Q_N^D \\ &+ vf_N(1 - Q_N^D) - f_O(1 - Q_N^D) + f_O(1 - Q_O^D - Q_N^D) - C_N = 0 \end{aligned}$$

while for $Q_O^D = 0$ it is necessary and sufficient that:

(A9a) $$f_O(1 - Q_N^D) - C_O \leq 0$$

(A9b) $$-vf_N'(1 - Q_N^D)\, Q_N^D + vf_N(1 - Q_N^D) - C_N = 0$$

Under our assumptions, (A8a)-(A8b) and (A9a)-(A9b) define downward-sloping and continuous reaction functions in the space of quantity pairs, and moreover these reaction functions have one and only one intersection; thus Q_O^D and Q_N^D are uniquely defined.

Innovation is drastic in this case if and only if $Q_O^D = 0$. Comparing (A5a) and (A9a), it follows that innovation is drastic under differentiated duopoly if and only if $Q_N^D \geq Q_N^L$, and

in the latter instance we have $Q_N^D = Q_N^C$ since (A9b) coincides with (A4b). Thus, innovation is drastic under differentiated duopoly precisely when it is drastic under dominant-fringe structure. Q.E.D.

Proof of Lemma. (a) The profit function of an old-profit monopolist who does not adopt the new product are:

$$\Pi = [f_O(1 - Q_O) - C_O]\, Q_O$$

and the unique maximizer Q_O^{OM} is defined by:

$$f_O'(1 - Q_O^{OM})\, Q_O^{OM} - f_O(1 - Q_O^{OM}) - C_O = 0 \tag{A10}$$

Since $\underline{v} < v \leq v^A$, Q_O^M and Q_N^M are defined by (A3a)-(A3b), and substituting $Q_O^M + Q_N^M = Q_O^{OM}$ into (A3a) gives (A10).

(b) Subtracting (A3a) from (A3b) gives a condition equivalent to (A6), with Q_N^M replacing $\underline{Q}_N^C$. Thus the result holds if (A6) determines Q_N^C, that is, the dominant firm does not choose the limit pricing strategy, when $\underline{v} < v \leq v^A$. Now, (A1a)-(A1b) imply $Q_N^M < Q_N^L$ when $v = v^A$, while subtracting (A5a) from (A5b) gives (A6) with Q_N^M replacing $\underline{Q}_N^C$; this establishes that Q_N^C is determined by (A6) at $v = v^A$. It is also true that Q_N^C is strictly increasing in v at $v = v^A$, for differentiation of (A6) gives:

$$\begin{aligned} \text{sign}\left[\frac{\partial Q_N^C}{\partial v}\right] &= \text{sign}[-f_N'(1 - Q_N^C)\, Q_N^C + f_N(1 - Q_N^C)] \\ &= \text{sign}[-f_N'(1 - Q_N^M)\, Q_N^M + f_N(1 - Q_N^M)] > 0 \end{aligned} \tag{A11}$$

where the inequality follows from (A1b). Moreover, the second term in (A11) remains positive for smaller Q_N^C, so that Q_N^C continues to be an increasing function of v at lower levels of v; thus $Q_N^C < Q_N^L$ for all $\underline{v} < v < v^A$, and consequently Q_N^C is determined by (A6). Q.E.D.

Proof of Proposition 2. It remains to consider the case of $v > v^A$. Note first that $Q_N^M < Q_N^C$ for $v^A < v < v^B$: this may be seen by subtracting (A4a), which holds with strict inequality, from (A4b) and comparing with (A6), when $Q_N^C < Q_N^L$; and by comparing (A4b) with (A5b), using (A7), when $Q_N^C = Q_N^L$. Further, $Q_N^C = Q_N^M$ for $v \geq v^B$. Let us now make the dependence on v explicit by writing Π^C and Π^M as $\Pi^C(v)$ and $\Pi^M(v)$. We have:

$$\begin{aligned}
&\Pi^C(v) - (\Pi^M(v) - \Pi^{OM}) \\
&= \Pi^C(v^A) - (\Pi^M(v^A) - \Pi^{OM}) + \int_{v^A}^{v} \left[\frac{\partial\Pi^C(t)}{dt} - \frac{\partial\Pi^M(t)}{dt}\right]dt \\
&= \int_{v^A}^{\min\{v,v^B\}} [f_N(1 - Q_N^C)\, Q_N^C - f_N(1 - Q_N^M)\, Q_N^M]dt > 0
\end{aligned}$$

using $Q_N^M < Q_N^C \leq Q_N^L$ over the relevant interval, together with the fact that $f_N(1 - Q_N)\, Q_N$ is strictly increasing in Q_N for $Q_N \leq Q_N^L$, which follows from (A7). Q.E.D.

The proofs of Propositions 3 and 4 make use of the following lemmata:

Lemma A4. If $v^A < v < v^B$, then $Q_O^D < Q_N^C$.

Proof. If $Q_N^C < Q_N^L$, we have, using (A6):

$$-vf'_N(1 - Q_N^C)\, Q_N^C + vf_N(1 - Q_N^C) - C_N$$
$$= -f'_O(1 - Q_N^C)\, Q_N^C + f_O(1 - Q_N^C) + C_O$$

Q_N^M is determined by (A4b), and from the proof of Proposition 2 we know $Q_N^M < Q_N^C$; thus:

$$-vf'_N(1 - Q_N^C)\, Q_N^C + vf_N(1 - Q_N^C) - C_N < 0$$

and so:

$$-f'_O(1 - Q_N^C)\, Q_N^C + f_O(1 - Q_N^C) + C_O < 0$$

$Q_O^D \geq Q_N^C$ would then imply:

$$-f'_O(1 - Q_O^D - Q_N^D)\, Q_O^D + f_O(1 - Q_O^D - Q_N^D) - C_O < 0$$

which contradicts (A8a). If $Q_N^C = Q_N^L$, then the result follows from $Q_O^D + Q_N^D < Q_N^C$, which uses (A5a) and (A8a). Q.E.D.

Lemma A5. If $\underline{v} < v < v^B$, then $\text{sign}[Q_N^C - Q_N^D] = \text{sign}[Q_N^D - Q_O^D]$.

Proof. Suppose first that $Q_N^C < Q_N^L$. Subtracting (A8a) from (A8b) gives:

$$-[vf'_N(1 - Q_N^D) - f'_O(1 - Q_N^D)]\, Q_N^D + vf_N(1 - Q_N^D) - f_O(1 - Q_N^D)$$
$$+ C_O - C_N + f'_O(1 - Q_O^D - Q_N^D)\, [Q_O^D - Q_N^D] = 0$$

If $Q_O^D > Q_N^D$, then the last term on the left-hand side of this expression is strictly positive.

Since the remaining terms of the left-hand side have the same form as (A6), it follows that (A6) evaluated at Q_N^D is strictly negative when $Q_O^D > Q_N^D$, and so $Q_N^C < Q_N^D$; similarly for $Q_O^D \leq Q_N^D$. If $Q_N^C = Q_N^L$, then $Q_N^D < Q_N^C$ follows at once from $Q_O^D + Q_N^D < Q_N^C$. To show $Q_N^D > Q_O^D$, let v′ denote the smallest v such that $Q_N^C = Q_N^L$; we have $Q_N^D < Q_N^C$ at v = v′, so $Q_N^D < Q_N^C$ for v slightly below v′. Thus $Q_O^D < Q_N^D$ holds for v slightly below v′, and since Q_N^D is strictly increasing in v and Q_O^D strictly decreasing in v for $\underline{v} < v < v^B$, $Q_O^D < Q_N^D$ continues to hold for larger v. Q.E.D.

Proof of Proposition 3. It is convenient in this instance to consider parameterization by C_O rather than v. Thus we fix v and define $\underline{C}_O$ by:

$$vf_N(1) - C_N = f_O(1) - \underline{C}_O$$

Viability of the new product requires $C_O > \underline{C}_O$. By arguments analogous to those of Lemmas A1-A3 it follows that there exist C_O^A and C_O^B, with $\underline{C}_O < C_O^A < C_O^B$, such that innovation is nondrastic under joint monopoly if and only if $\underline{C}_O < C_O < C_O^A$, and nondrastic under dominant-fringe structure and differentiated duopoly if and only if $\underline{C}_O < C_O < C_O^B$. We complete the proof by considering the region $C_O^A < C_O < C_O^B$ (corresponding to $v^A < v < v^B$). Making explicit the dependence of profits on C_O, we may write:

$$\text{(A12)} \qquad \Pi^M - \Pi^{OD}(C_O) - \Pi^C(C_O) = \{\Pi^M - \Pi^{OD}(C_O^B) - \Pi^C(C_O^B)\} + \int_{C_O}^{C_O^B} \left[\frac{\partial \Pi^{OD}}{\partial C_O} + \frac{\partial \Pi^C}{\partial C_O}\right] dC_O$$

where Π^M is independent of C_O due to $C_O > C_O^A$. The term in braces vanishes since

innovation is drastic under all three market structures when $C_O \geq C_O^B$. To ease notation we will use a lower bar to indicate that a function is evaluated at $1 - Q_O^D - Q_N^D$, and an upper bar to indicate that a function is evaluated at $1 - Q_N^D$. The result follows if the integrand in (A12) is positive; this integrand may be written:

$$\text{(A13)} \qquad \frac{\partial \Pi^{OD}}{\partial C_O} + \frac{\partial \Pi^{C}}{\partial C_O} = -\underline{f}'_O\, Q_O^D \frac{\partial Q_N^D}{\partial C_O} + Q_N^D$$

$$= \frac{1}{\Delta} \{ -\underline{f}'_O\, Q_O^D\, [-\underline{f}''_O\, Q_N^D + \underline{f}'_O] + \Delta Q_N^C \}$$

where Δ, which derives from the reaction functions under differentiated duopoly, is given by:

$$\Delta \equiv -\underline{f}''_O\, \underline{f}'_O\, [Q_O^D + Q_N^D] + 3[\underline{f}'_O]^2 +$$
$$[\underline{f}''_O\, Q_O^D - 2\underline{f}'_O] \cdot \{ [v\bar{f}''_N - \bar{f}''_O]\, Q_N^D - 2[v\bar{f}'_N - \bar{f}'_O] \} > 0$$

The term in braces in (A13) may be written:

$$\text{(A14)} \qquad \underline{f}''_O\, \underline{f}'_O\, Q_O^D\, Q_N^D - [\underline{f}'_O]^2 Q_O^D - \underline{f}''_O\, \underline{f}'_O\, [Q_O^D + Q_N^D]\, Q_N^C + 3[\underline{f}'_O]^2 Q_N^C$$
$$+ \{ \underline{f}''_O\, Q_O^D - 2\underline{f}'_O \} \cdot \{ [v\bar{f}''_N - \bar{f}''_O]\, Q_N^D - 2[v\bar{f}'_N - \bar{f}'_O] \}\, Q_N^C$$

The last term in (A14), which includes the product of expressions in braces, is strictly positive, and the remaining terms may be written:

$$\text{(A15)} \qquad \underline{f}''_O\, \underline{f}'_O\, Q_O^D\, [Q_N^D - Q_N^C] + [\underline{f}'_O]^2 [3Q_N^C - Q_O^D] - \underline{f}''_O\, \underline{f}'_O\, Q_N^D\, Q_N^D$$

The third term in (A15) is evidently positive. By Lemma A4 we have $Q_N^C > Q_O^D$ and the second term is strictly positive. If $Q_N^D \geq Q_N^C$, then we have $Q_O^D \geq Q_N^D$, by Lemma A5, but these inequalities contradict $Q_N^C > Q_O^D$; thus $Q_N^D < Q_N^C$ and the first term is positive. Q.E.D.

Proof of Proposition 4. We may write:

$$\text{(A16)} \qquad \Pi^{ND}(v) - \Pi^{C}(v) = \Pi^{ND}(\underline{v}) - \Pi^{C}(\underline{v}) + \int_{\underline{v}}^{v} \left[\frac{\partial \Pi^{ND}}{\partial v} - \frac{\partial \Pi^{C}}{\partial v} \right] dv$$

Since $\Pi^{ND}(\underline{v}) = \Pi^{C}(\underline{v}) = 0$, we must show that the integrand in (A16) is strictly positive:

$$\text{(A17)} \qquad \frac{\partial \Pi^{ND}}{\partial v} - \frac{\partial \Pi^{C}}{\partial v} = f_N(1 - Q_N^D)\, Q_N^D - f_O(1 - Q_O^D - Q_N^D)\, Q_N^D \frac{\partial Q_O^D}{\partial v} - f_N(1 - Q_N^C)\, Q_N^C$$

Since Q_O^D is strictly decreasing in v, the second term on the right-hand side of (A17) is strictly positive. Further, it was established above that $f_N(1 - Q_N)\, Q_N$ is strictly increasing in Q_N for $Q_N \leq Q_N^L$, so the result holds if $Q_N^D \geq Q_N^C$. Now, there is a v′ such that $Q_O^D \geq Q_N^D$ for $\underline{v} < v \leq v'$, and thus by Lemma A5 we have $Q_N^D \geq Q_N^C$ on this interval.

Next consider parameterization by C_O as in Proposition 3:

$$\text{(A18)} \qquad \Pi^{ND}(C_O) - \Pi^{C}(C_O) = \Pi^{ND}(C_O^B) - \Pi^{C}(C_O^B) - \int_{C_O}^{C^B} \left[\frac{\partial \Pi^{ND}}{\partial C_O} - \frac{\partial \Pi^{C}}{\partial C_O} \right] dC_O$$

The integrand in (A18) must be strictly negative. Using lower bars to denote evaluation at $1 - Q_O^D - Q_N^D$ and upper bars to denote evaluation at $1 - Q_N^D$, we have:

$$\text{(A19)} \qquad \frac{\partial \Pi^{ND}}{\partial C_O} - \frac{\partial \Pi^{C}}{\partial C_O} = -\underline{f}_O' \, Q_N^D \frac{\partial Q_O^D}{\partial C_O} - Q_N^C$$

$$= \frac{1}{\Delta} \{ -\underline{f}_O' \, Q_N^D \, ([v\bar{f}_N'' - \bar{f}_O'' + \underline{f}_O''] \, Q_N^D - 2[v\bar{f}_N' - \bar{f}_O' + \underline{f}_O']) - \Delta Q_N^C \}$$

where Δ is defined in the proof of Proposition 3. The expression in braces in (A19) may be written:

$$\{ -\underline{f}_O'' \, \underline{f}_O' \, Q_N^D + 2[\underline{f}_O']^2 - \underline{f}_O' \, ([v\bar{f}_N'' - \bar{f}_O''] \, Q_N^D - 2[v\bar{f}_N' - \bar{f}_O']) \} \cdot [Q_N^D - Q_N^C]$$
$$- \{ -\underline{f}_O'' \, \underline{f}_O' \, Q_O^D + [\underline{f}_O']^2 + [\underline{f}_O'' \, Q_O^D - \underline{f}_O'] \cdot ([v\bar{f}_N'' - \bar{f}_O''] \, Q_N^D - 2[v\bar{f}_N' - \bar{f}_O']) \} \, Q_N^C$$

which is strictly negative as long as $Q_N^D < Q_N^C$. By Lemma A5 we know that the latter is true for $C_O' < C_O < C_O^B$, for some C_O'. Q.E.D.

Proof of Proposition 5. Social surplus when the new product has been adopted by a joint monopolist is:

$$S_A^M = \max\{Q_O^{OM} - Q_N^M, 0\} \int_{1-Q_O^{OM}}^{1-Q_N^M} (f_O(\omega) - C_O) d\omega + \int_{1-Q_N^M}^{1} (vf_N(\omega) - C_N) d\omega$$

It can be shown using (3a) and (3b) that Q_N^M is strictly increasing in v for $\underline{v} < v \leq v^A$, while we have already noted that Q_N^M is strictly increasing in v for $v > v^A$; thus S_A^M is strictly

increasing in v. Social surplus under competition in the old product market when the new product has not been adopted is given by:

$$S_{NA}^{C} = \int_{1-Q_N^L}^{1} (f_O(\omega) - C_O)d\omega$$

where Q_N^L is given by (A5a). Now, $Q_N^M \to 0$ as $v \to \underline{v}$, while $Q_N^L > Q_O^{OM}$ follows from (A5a and (A10); thus $S_{NA}^C > S_A^M$ for v close to $\underline{v}$. At $v = v^B$, in contrast, we have $Q_N^M = Q_N^L$, from (A4b) and (A5b), and moreover (A4a) and (A4b) imply:

$$vf_N(1 - Q_N^L) - C_N > f_O(1 - Q_N^L) - C_O$$

Thus $S_A^M > S_{NA}^C$ for $v = v^B$. It follows that $S_A^M = S_{NA}^C$ for a unique v^* satisfying $\underline{v} < v^* < v^B$, with $S_A^M < S_{NA}^C$ for $v < v^*$ and $S_A^M > S_{NA}^C$ for $v > v^*$. Q.E.D.

Appendix 2

Robustness of the Equality of Returns

In this appendix we demonstrate that the equality of returns under competition and protected monopoly is robust, in the sense that for appropriate perturbations of utility functions, either structure may yield strictly greater returns than the other. First, for given Q_O and Q_N, the utility of consumer ω_N, which will be denoted U_N, is given by:

$$U_N = f_O(1 - Q_N) - f_O(1 - Q_O - Q_N)$$

Since U_N is strictly increasing in Q_O, we can recast the joint monopolist's profit maximization problem in terms of Q_N and U_N, and there will be unique maximizers Q_N^M and U_N^M. In the dominant-fringe case, U_N is given by:

$$U_N = f_O(1 - Q_N) - C_O$$

Again U_N is increasing in Q_N, so we can think of the new-product monopolist as choosing a profit-maximizing level U_N^C. It is easy to see that $U_N^C > U_N^M$ when $v < v^A$.

We now consider perturbations of the utility function that have the form $g(vf_N(\omega) - P_N)$, where $g' > 0$ and $g(U) \geq U$ for all U. The utility of the old product is held fixed, which ensures that Π^{OM} is unaffected by the perturbation. Now fix a constant X with $U_N^M < X < U_N^C$, and suppose $g(U) > U$ if and only if $U < X$. If the joint monopolist chooses Q_N^M and U_N^M in the same way as before the perturbation, then P_O is unchanged if the perturbation is slight, while P_N becomes strictly greater; thus the maximized profit level is strictly greater following the perturbation. Under dominant-fringe structure, in contrast, choosing U_N^C gives the same level of profits, since utility is unaffected for $U \geq X$, while offering utility levels U

$< X$ will continue to be unattractive if the perturbation is sufficiently slight; thus Π^C is unaffected. It follows that we have $\Pi^M - \Pi^{OM} > \Pi^C$ following the perturbation, and returns are strictly greater under protected monopoly. By a symmetric argument, it follows that competition will provide strictly greater returns following perturbations such that $g(U) > U$ if and only if $U > X$.

We can interpret the perturbations in terms of cross-product interactions obtaining from the new product when utility is low, whereby consuming the new product boosts the utility derived from the consumer's entire consumption bundle.

References

Aoki, R. [1991], "R&D Competition for Product Innovation: An Endless Race," *American Economic Review* 81, 252-256.

Arrow, K.J. [1962], "Economic Welfare and the Allocation of Resources for Invention," in NBER, *The Rate and Direction of Innovative Activity* (Princeton, N.J.: Princeton Univ. Press).

Aghion, P., and P. Howitt [1990], "A Model of Growth through Creative Destruction," NBER Working Paper No. 3223.

Attack, J., and J.K. Breuckner [1982], "Steel Rails and American Railroads, 1867-1880," *Explorations in Economic History* 19, 339-359.

Bagwell, K., and R.W. Staiger [1990], "Risky R&D in Oligopolistic Product Markets," CMSEMS D.P. No. 872, Northwestern University.

Bain, J.S. [1949], "A Note on Pricing in Monopoly and Oligopoly," *American Economic Review* 39, 448-464.

Beath, J., Y. Katsoulacos and D. Ulph [1987], "Sequential Product Innovation and Industry Evolution," *Economic Journal* 97, 32-43.

Brozen, Y. [1951], "Invention, Innovation and Imitation," *American Economic Review* 41, 239-257.

Dasgupta, P., and J.E. Stiglitz [1980], "Uncertainty, Industrial Structure and the Speed of R&D," *Bell Journal of Economics* 11, 1-28.

DeBrock, L.M., and R.T. Masson [1985], "Sooner or Later? Inventive Rivalry and Welfare," *International Journal of Industrial Organization* 3, 421-438.

Dutta, P.K., S. Lach amd A. Rustichini [1990], "Innovation and Product Differentiation," CMSEMS D.P. Np. 894, Northwestern University.

Fisher, McKie and Mancke [1983], *IBM and the U.S. Data Processing Industry: An Economic History* (Praeger Publications).

Gabszewicz, J., and J.-F. Thisse [1979], "Price Competition and Income Disparities," *Journal of Economic Theory* 20, 340-359.

——— and ——— [1980], "Entry (and Exit) in a Differentiated Industry," *Journal of Economic Theory* 22, 327-338.

Gilbert, R.J., and D.M.G. Newbery [1982], "Preemptive Patenting and the Persistence of Monopoly," *American Economic Review* 72, 514-526.

Greenstein, S., and G. Ramey [1988], "Product Inertia and the Incentive to Innovate," CEPR Publication No. 149, Stanford University.

Grossman, G.M., and E. Helpman [1991a], "Quality Ladders and Product Cycles," *Quarterly Journal of Economics* 106, 557-586.

——— and ——— [1991b], "Quality Ladders in the Theory of Growth," *Review of Economic Studies* 58, 43-61.

Kamien, M.I., and N.L. Schwartz [1982], *Market Structure and Innovation* (Cambridge: Cambridge Univ. Press).

Lange, O. [1943], "A Note on Innovations," *Review of Economics and Statistics* 25, 19-25.

Loury, G.C. [1979], "Market Structure and Innovation," *Quarterly Journal of Economics* 93, 395-410.

Panati, C. [1987], *Extraordinary Origins of Everyday Things* (New York: Harper and Row).

Ramey, G. [1988], "Preemption versus Learning in Product Innovation," Discussion Paper #88-25, Department of Economics, UC San Diego.

Scherer, F.M. [1980], *Industrial Market Structure and Economic Performance*, second edition (Chicago: Rand McNally).

Schumpeter, J.A. [1975], *Capitalism, Socialism and Democracy* (New York: Harper and Row).

Scotchmer, Suzanne, and Jerry Green [1990], "Novelty and Disclosure in Patent Law," *Rand Journal of Economics* 21, 131-146.

Shaked, A., and J. Sutton [1982], "Relaxing Price Competition through Product Differentiation," *Review of Economic Studies* 49, 3-13.

——— and ——— [1983], "Natural Oligopolies," *Econometrica* 51, 1469-1484.

——— and ——— [1990], "Multiproduct Firms and Market Structure," *RAND Journal of Economics* 21, 45-62.

Stokey, N. [1988], "Learning by Doing and the Introduction of New Goods," *Journal of Political Economy* 96, 701-717.

——— [1991], "Human Capital, Product Quality, and Growth," *Quarterly Journal of Economics* 106, 587-616.

Temin, P. [1964], *Iron and Steel in Nineteenth Century America: An Economic Inquiry* (Cambridge, Mass.: MIT Press).

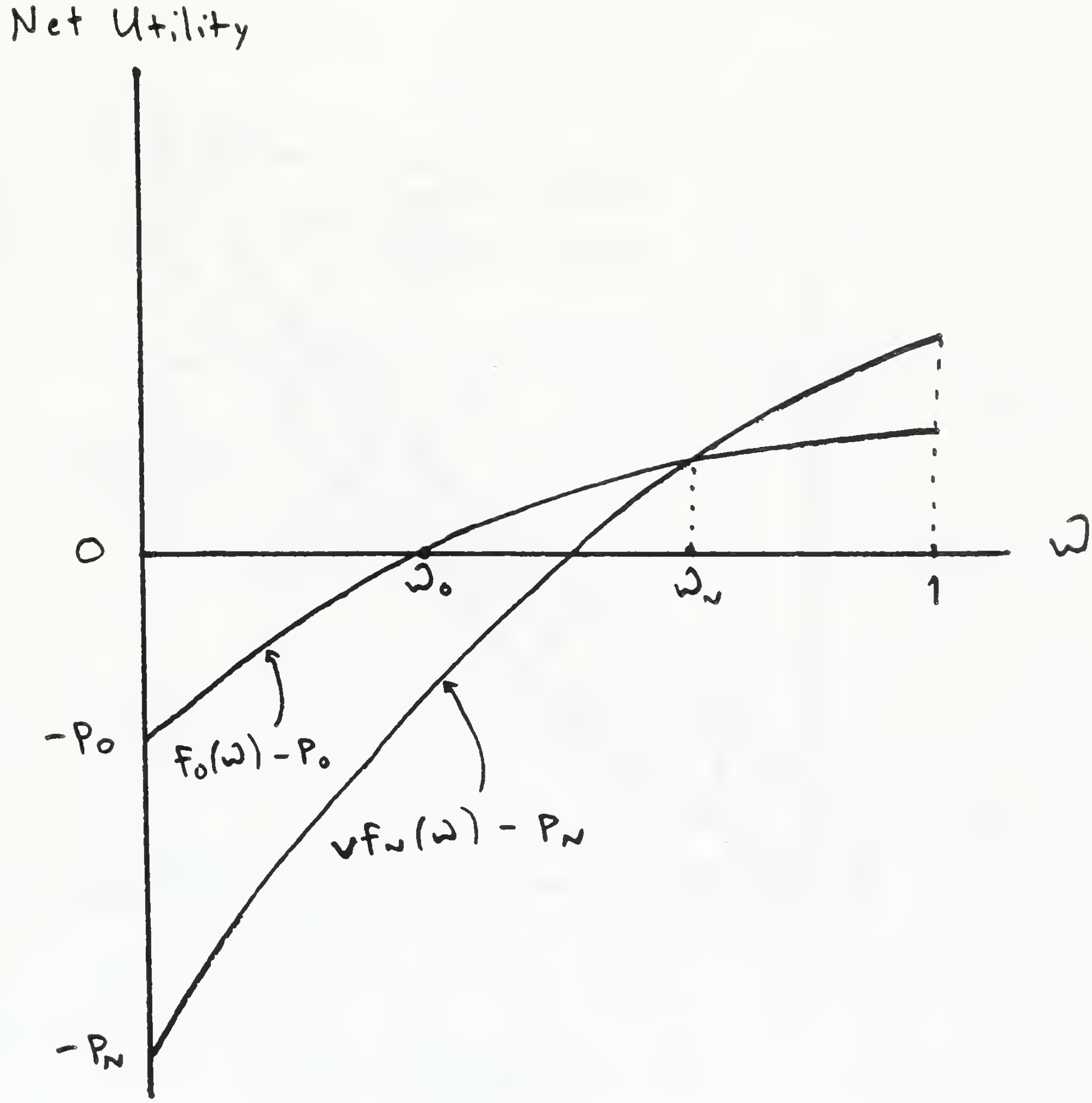

Figure 1

Net Utility from Product O and Product N under prices P_O and P_N

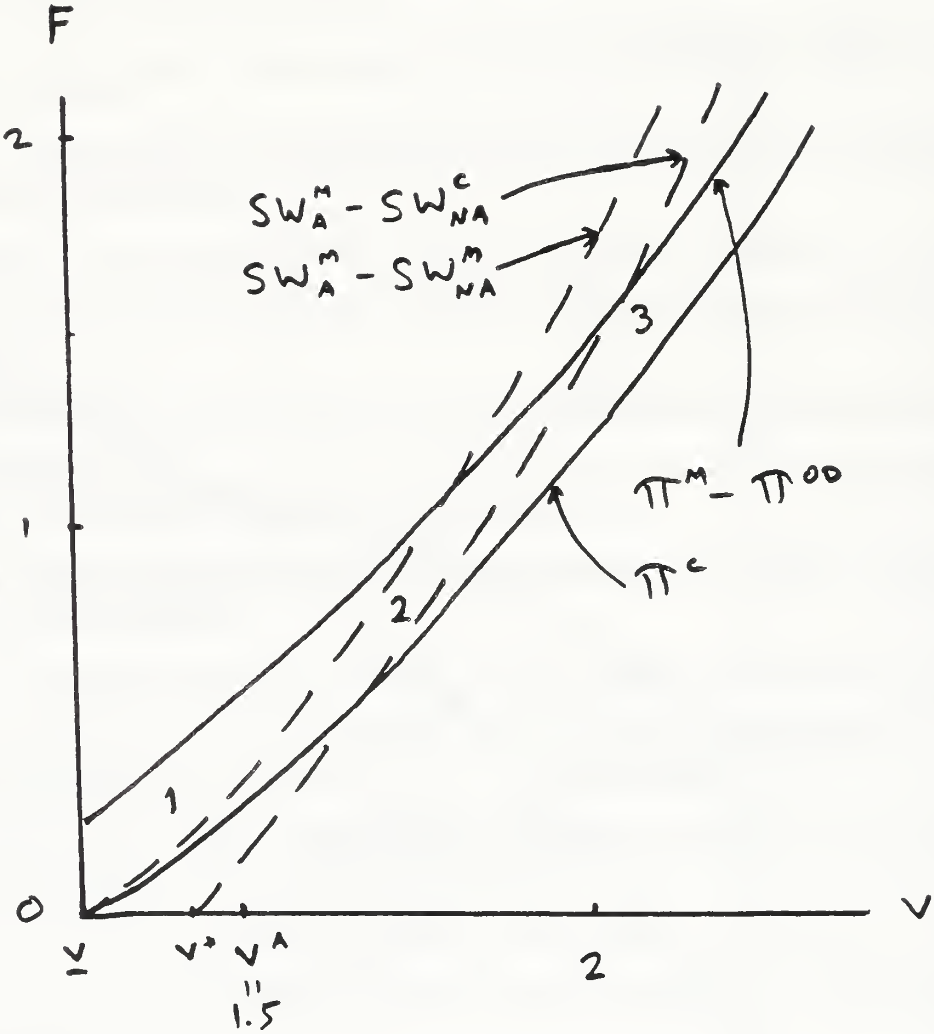

Figure 2

Social Surplus with Fixed R&D Cost F

Regions 1, 2 and elsewhere - Competition gives greater social surplus

Region 3 - Threatened monopoly gives greater social surplus

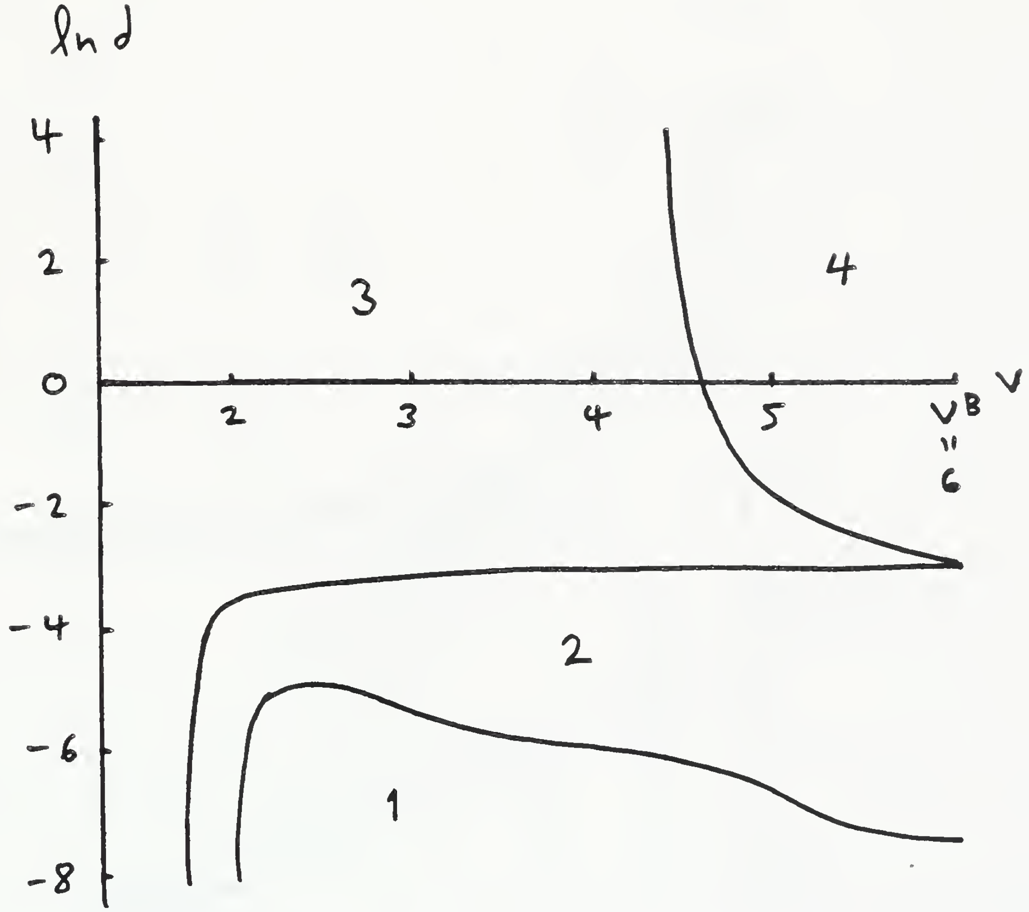

Figure 3

Social Surplus in Patent Race Equilibria
Case of Disintegrated R&D

Region 1 - Threatened > Competitive > Protected
Region 2 - Competitive > Threatened > Protected
Region 3 - Competitive > Protected > Threatened
Region 4 - Protected > Competitive > Threatened

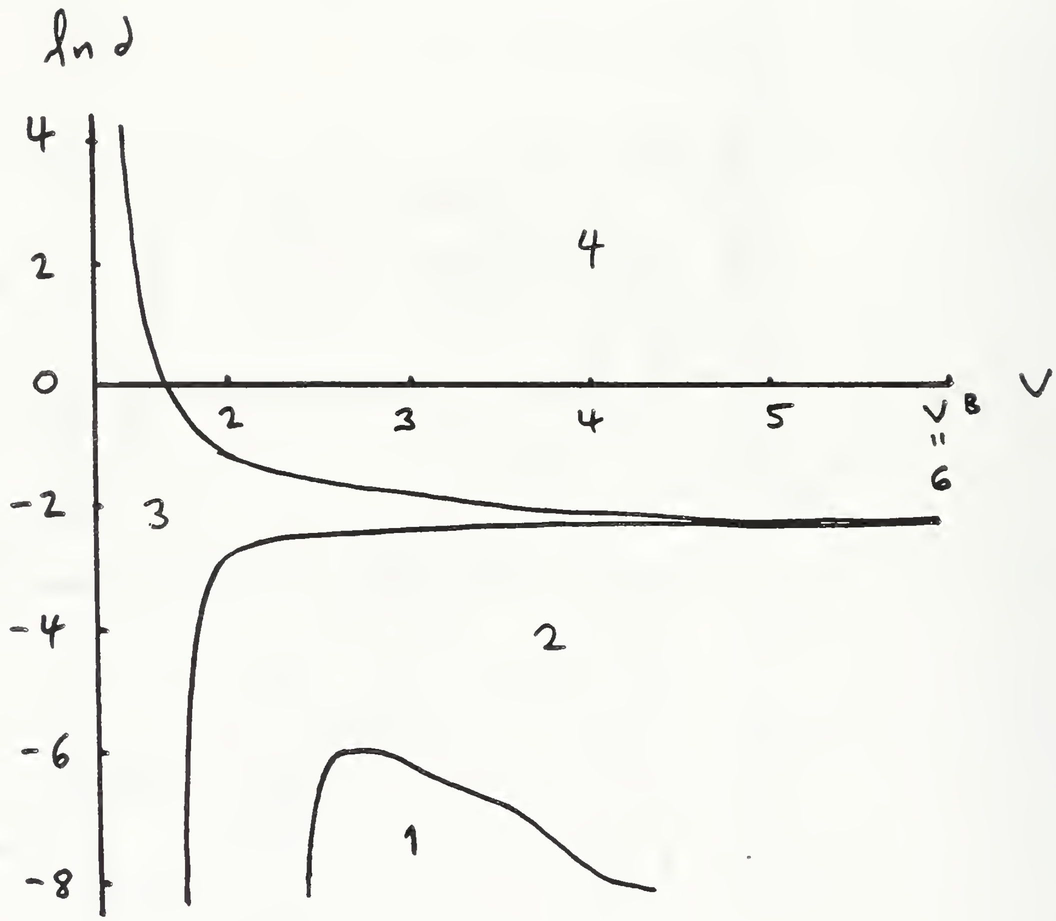

Figure 4

Social Surplus in Patent Race Equilibria
Case of Integrated R & D

Regions defined as in Figure 4

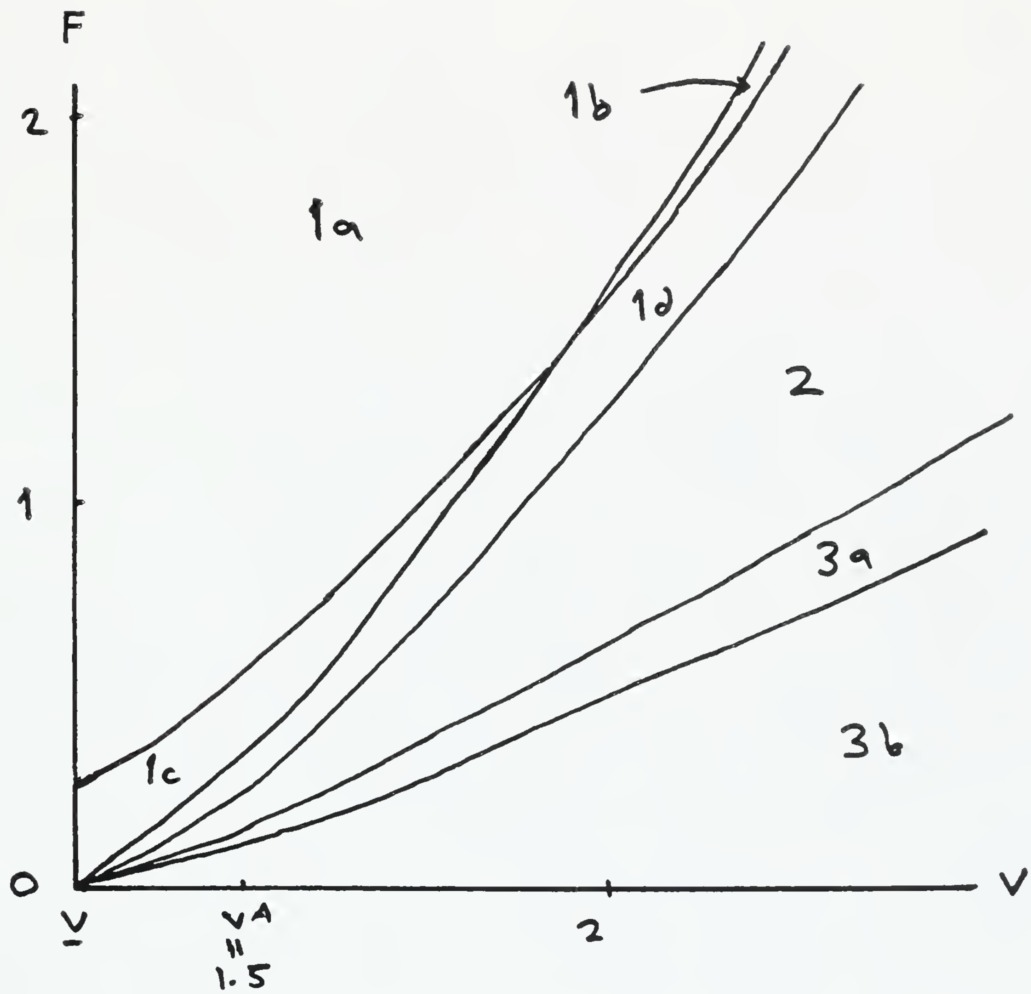

Figure 5

Duopoly Adoption Equilibria

Regions 1a-d - Neither firm adopts

Region 2 - One firm adopts

Regions 3a-b - Both firms adopt